On Handling the Inverse Analysis of Passive Spectroscopic Cargo Monitoring Data

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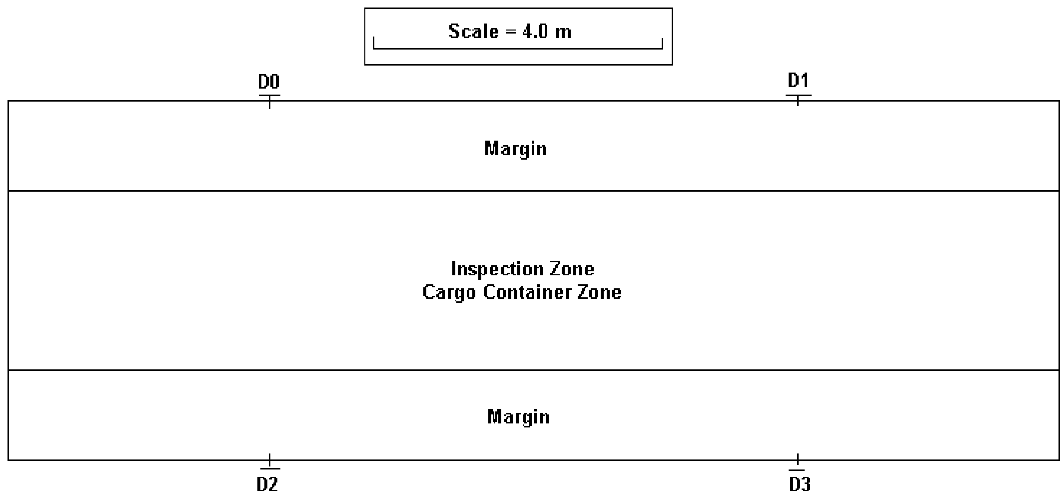
Introduction

- Passive gamma spec is costly as a cargo monitoring technology, and it has serious limitations.
- Possibly more useful when combined with other technologies like neutron counting
  - But then again, a neutron alarm alone should be enough to warrant more invasive action
- Cost-benefit analysis is controversial

Objectives

- Apply Monte Carlo Library Least Squares
- Use few to no high-resolution spectrometers
- Source strength ID
  - ‘radioisotope weigh station’
  - resistant to intentional shielding
- Source location
  - Security need
- Demonstrate new methodology

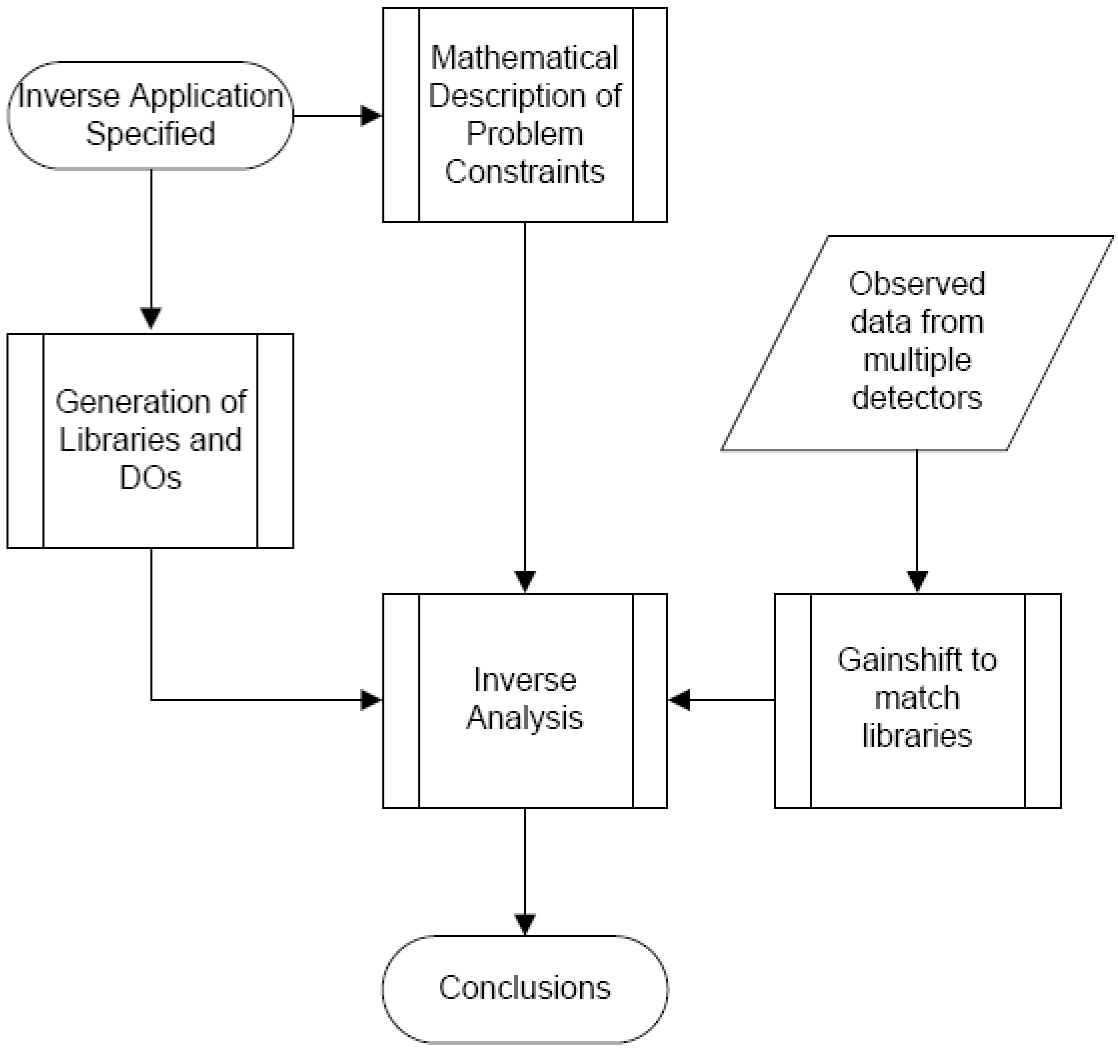
Example Schematic



2-D approximation of secondary screening of 48' semi-tractor trailer by quad-panel Advanced Spectroscopic Portal

Inter-comparison of MCLLS results for separate detectors at known positions yields most likely source position, activity, and shielding configurations.

Overview



Parameterize sources of variability to model detector response

Impose as much constraint as possible

Express parameterization in 3d transport simulation, estimating reference spectra and DOs

Process measurement data for comparison to simulation

Then, conduct lin/non-linear searches through parameter space and apply constraints as appropriate

Second degree expansion

- In three variables, types of shielding
- With a scalar multiplier

$$\vec{f}(\vec{x}+\Delta\vec{x}) = \alpha\left[\overrightarrow{f_1}(\vec{x}_0) + \frac{\partial\overrightarrow{f_1}}{\partial x_1}\Delta x_1 + \frac{\partial\overrightarrow{f_1}}{\partial x_2}\Delta x_2 + \frac{\partial\overrightarrow{f_1}}{\partial x_3}\Delta x_3 + \frac{1}{2}\frac{\partial^2\overrightarrow{f_1}}{\partial {x_1}^2}\Delta {x_1}^2 + \frac{1}{2}\frac{\partial^2\overrightarrow{f_1}}{\partial {x_2}^2}\Delta {x_2}^2 + \frac{1}{2}\frac{\partial^2\overrightarrow{f_1}}{\partial {x_3}^2}\Delta {x_3}^2 \right.$$
$$+\left(\frac{\partial^2\overrightarrow{f_1}}{\partial x_1\partial x_2} + \frac{\partial^2\overrightarrow{f_1}}{\partial x_2\partial x_1}\right)\Delta x_1\Delta x_2 + \left(\frac{\partial^2\overrightarrow{f_1}}{\partial x_1\partial x_3} + \frac{\partial^2\overrightarrow{f_1}}{\partial x_3\partial x_1}\right)\Delta x_1\Delta x_3$$
$$\left. +\left(\frac{\partial^2\overrightarrow{f_1}}{\partial x_2\partial x_3} + \frac{\partial^2\overrightarrow{f_1}}{\partial x_3\partial x_2}\right)\Delta x_2\Delta x_3\right]$$

- References and derivatives estimated by MC simulation

Unraveling Location and Intensity

$$\alpha = c\frac{I}{\varepsilon_{i,k}(\mu_x)}$$

- Intensity
- Efficiency
- Cross-section

Location and Intensity

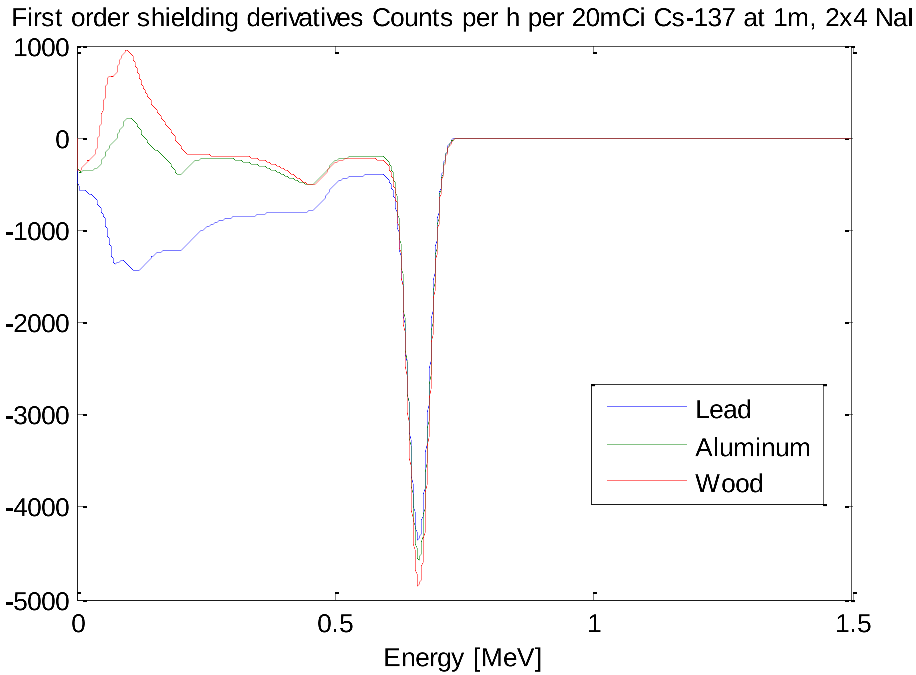
- Given a histogram probability distribution for path lengths through a detector centered at an origin as a continuous function of isotropic source position one may…

$$\overline{P}_{\ell\,i} = \frac{\iiint_{V_i} dV_i \; P_\ell(x,y,z)}{\iiint_{V_i} dV_i}$$

$$\frac{1}{\varepsilon_{i,k}(\mu_x)} \cong \sum_m e^{-\overline{P}_{\ell\,i,m}\ell_m\mu_x}$$

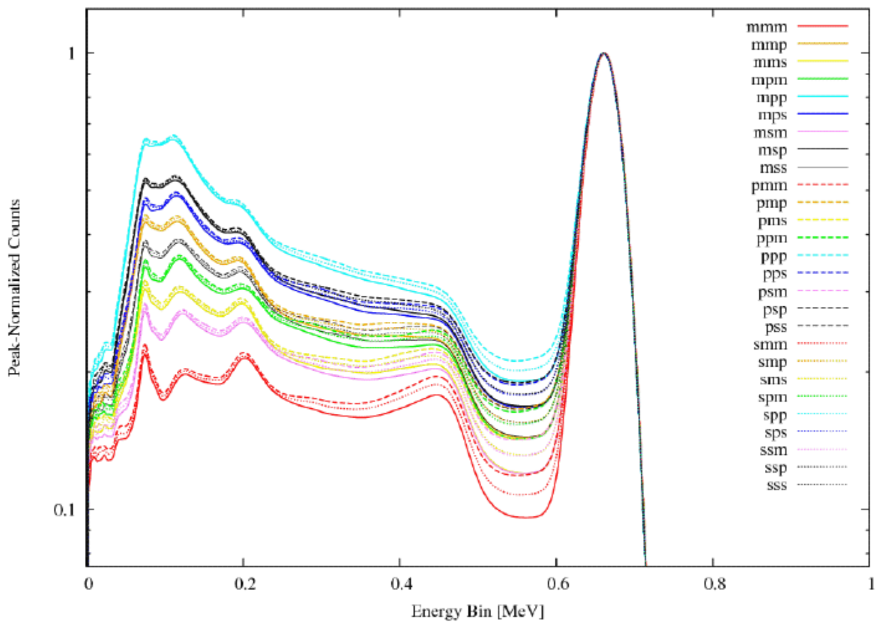
First Finite Differences

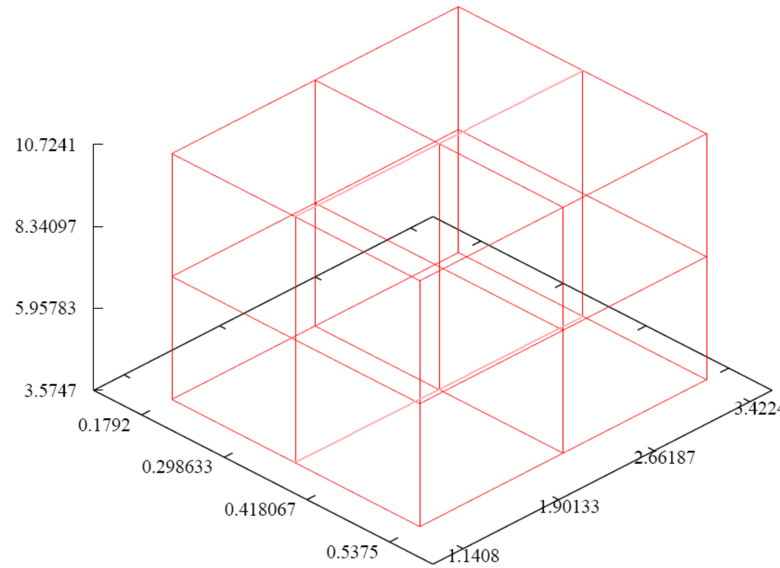
$$\frac{\partial \overrightarrow{f_1}}{\partial x_1} \cong \frac{\vec{f_1}(x_{1,p}, x_{2,s}, x_{3,s}) - \vec{f_1}(x_{1,m}, x_{2,s}, x_{3,s})}{2h_1}$$



	Pb [cm]	Al [cm]	Wood [cm]
50%	0.53754	3.42235	10.72411
Standard	0.358364	2.281566	7.1493867
-50%	0.179182	1.140782	3.57470333

$$\vec{f}(\vec{x} + \Delta\vec{x}) = \alpha \left[\overrightarrow{f_1}(\vec{x}_0) + \frac{\partial \overrightarrow{f_1}}{\partial x_1}\Delta x_1 + \frac{\partial \overrightarrow{f_1}}{\partial x_2}\Delta x_2 + \frac{\partial \overrightarrow{f_1}}{\partial x_3}\Delta x_3 \right]$$





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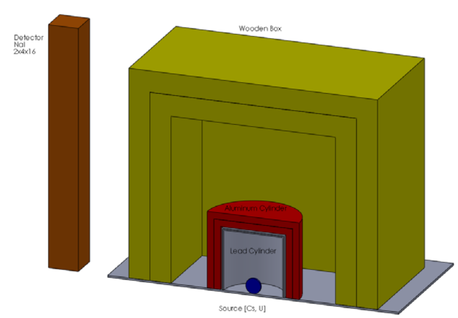
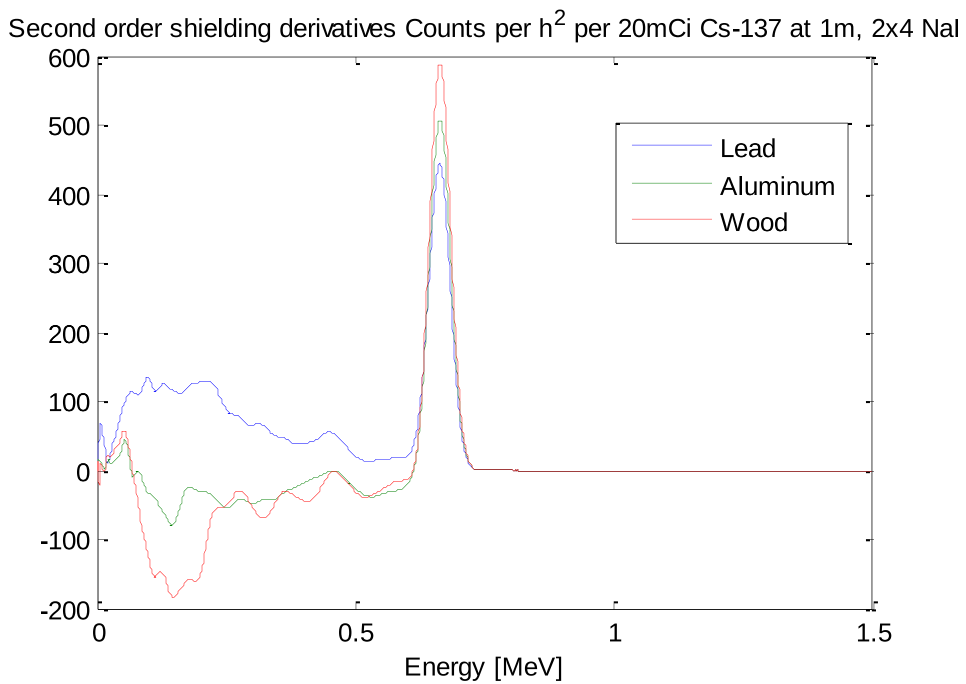


Second Finite Differences

$$\frac{\partial^2 \overrightarrow{f_1}}{\partial {x_1}^2} \cong \frac{\vec{f_1}(x_{1,p}, x_{2,s}, x_{3,s}) - 2\vec{f_1}(x_{1,s}, x_{2,s}, x_{3,s}) + \vec{f_1}(x_{1,m}, x_{2,s}, x_{3,s})}{{h_1}^2}$$



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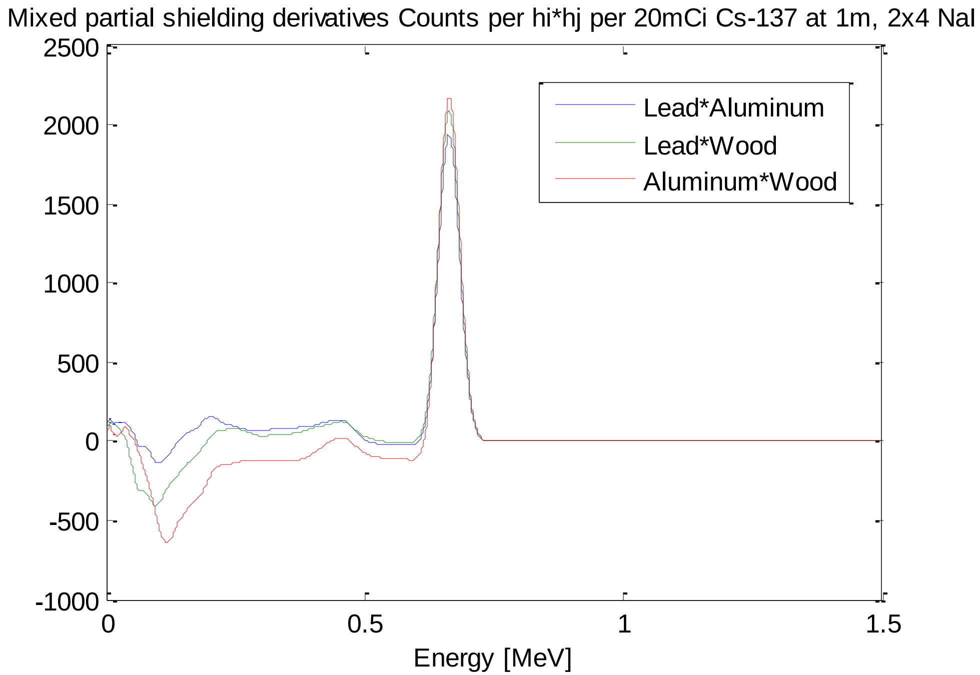
$$+ \left(\frac{\partial^2\overrightarrow{f_1}}{\partial x_1\partial x_2} + \frac{\partial^2\overrightarrow{f_1}}{\partial x_2\partial x_1}\right)\Delta x_1\Delta x_2 + \left(\frac{\partial^2\overrightarrow{f_1}}{\partial x_1\partial x_3} + \frac{\partial^2\overrightarrow{f_1}}{\partial x_3\partial x_1}\right)\Delta x_1\Delta x_3$$
$$+ \left(\frac{\partial^2\overrightarrow{f_1}}{\partial x_2\partial x_3} + \frac{\partial^2\overrightarrow{f_1}}{\partial x_3\partial x_2}\right)\Delta x_2\Delta x_3\Bigg]$$

Mixed Partial Finite Differences

$$\frac{\partial^2 \overrightarrow{f_1}}{\partial x_1 \partial x_2} \cong \frac{\vec{f_1}(x_{1,p}, x_{2,p}, x_{3,s}) - \vec{f_1}(x_{1,p}, x_{2,m}, x_{3,s}) - \vec{f_1}(x_{1,m}, x_{2,p}, x_{3,s}) + \vec{f_1}(x_{1,m}, x_{2,m}, x_{3,s})}{4 h_1 h_2}$$



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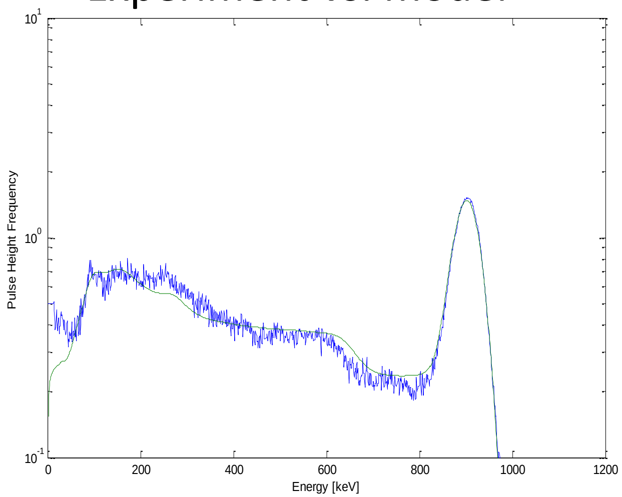
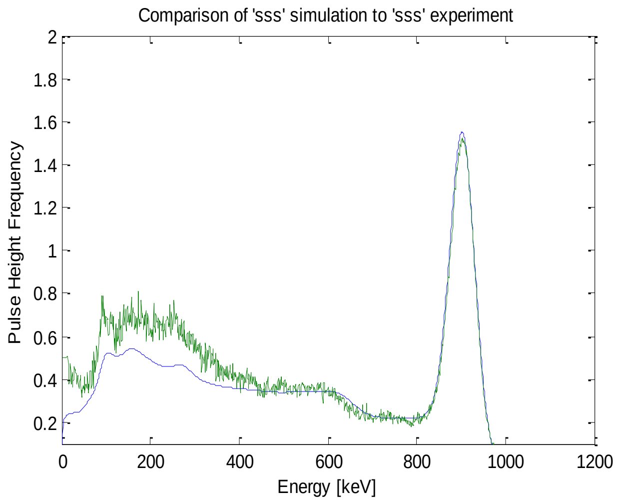
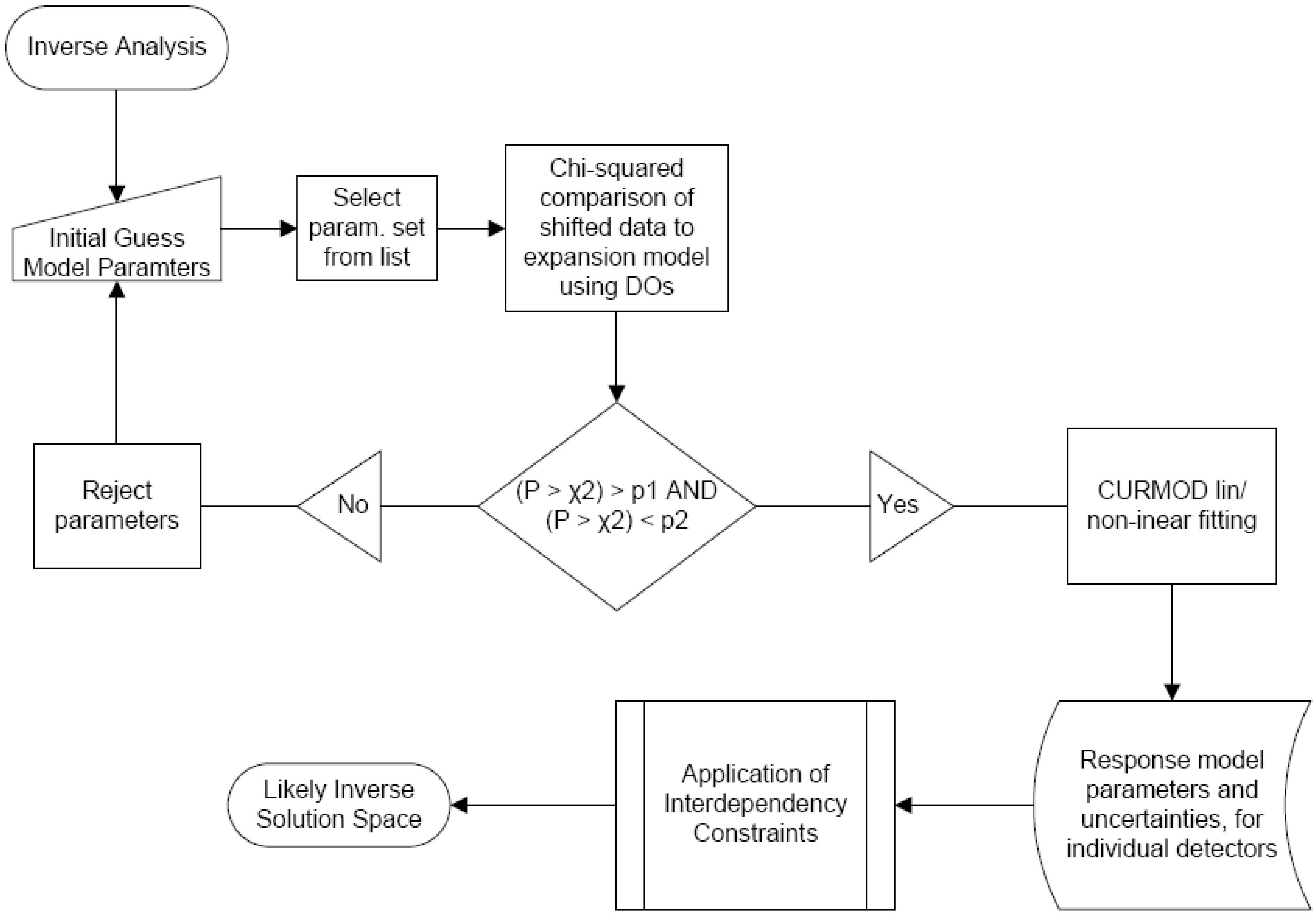
Example Results

Experiment vs. model

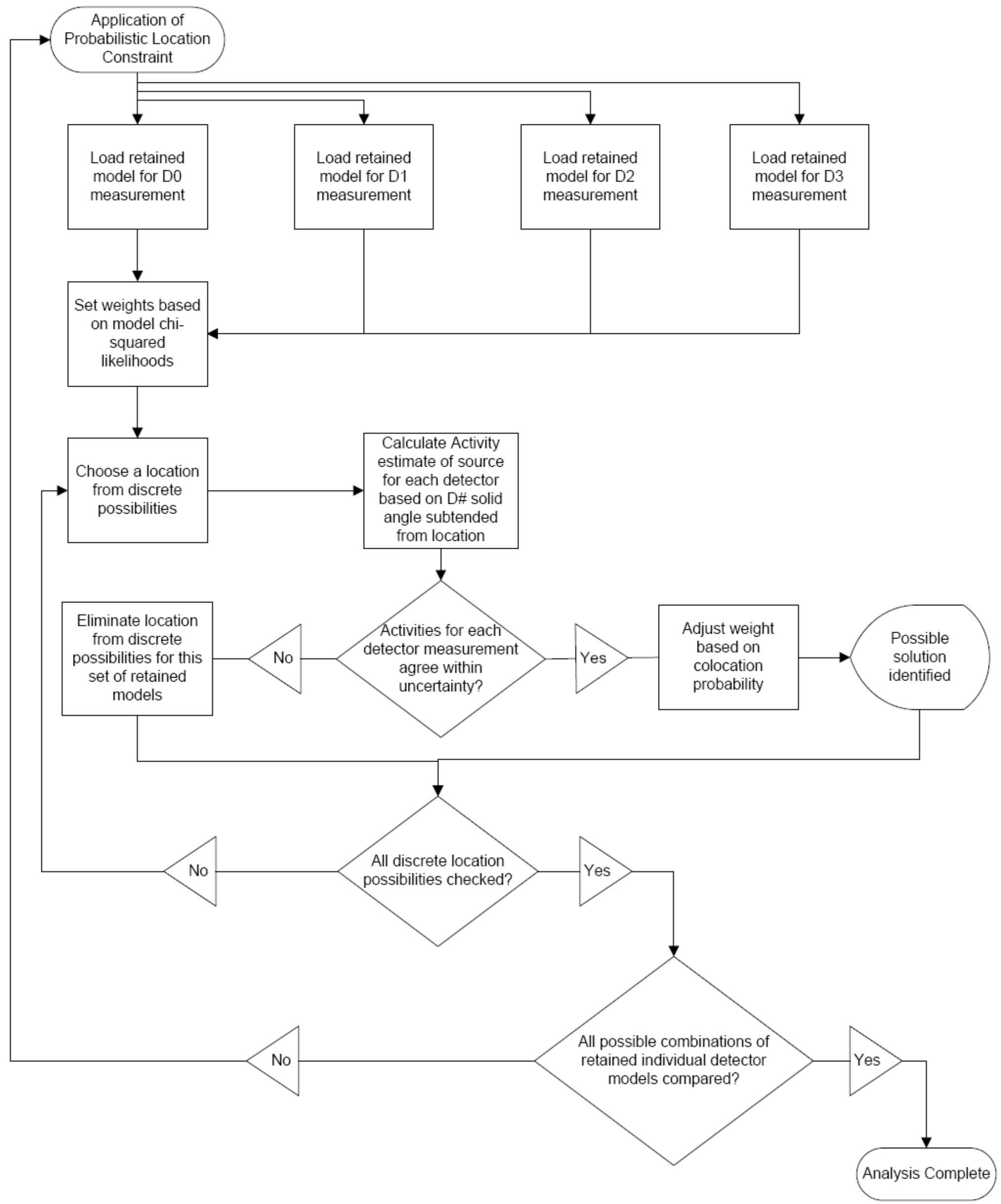




	Pb thickness	Al thickness	Wood thickness	Activity	# of iteration
Actual	0.358 cm	2.282 cm	7.149 cm	4 µCi	N/A
Calculated 2nd	0.27 cm	1.83 cm	11.10 cm	3.21 µCi	56
Calculated 1st O	0.10 cm	0.99 cm	14.47 cm	2.74 µCi	148



Inverse Analysis
Initial Guess Model Paramters
Select param. set from list
Chi-squared comparison of shifted data to expansion model using DOs
(P > χ2) > p1 AND (P > χ2) < p2
No
Yes
Reject parameters
CURMOD lin/ non-inear fitting
Response model parameters and uncertainties, for individual detectors
Application of Interdependency Constraints
Likely Inverse Solution Space



Application of Probabilistic Location Constraint
Load retained model for D0 measurement
Load retained model for D1 measurement
Load retained model for D2 measurement
Load retained model for D3 measurement
Set weights based on model chi-squared likelihoods
Choose a location from discrete possibilities
Calculate Activity estimate of source for each detector based on D# solid angle subtended from location
Eliminate location from discrete possibilities for this set of retained models
No
Activities for each detector measurement agree within uncertainty?
Yes
Adjust weight based on colocation probability
Possible solution identified
No
All discrete location possibilities checked?
Yes
No
All possible combinations of retained individual detector models compared?
Yes
Analysis Complete

Conclusion

- Passive gamma spec may have a decreasing role in cargo monitoring as it is costly and has limited impact for security, and is quite costly for the safety benefits it can produce.
- The MC LLS method has certain features that make it a viable platform for getting more out of a detection system.
  - Full spectrum analysis
  - Efficient linearization of L-M search algorithm
  - Highly customizable

Thank You!

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