SEMI-EMPIRICAL MODELING OF GAMMA-RAY DENSITY LOG WITH THE POSSIBILITY OF OBTAINING MORE INFORMATION

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- REVIEW OF EXISTING DENSITY LOG TREATMENT
- THE FORWARD SCATTER MODEL APPROACH
- MODEL VS. MONTE CARLO RESULTS
- DISCUSSION, FUTURE WORK, AND CONCLUSIONS

REVIEW OF EXISTING DENSITY LOG TREATMENT

The text by Ellis describes the existing treatment of the density log. It consists of:

- a "spine and ribs" treatment in which the "spine" represents the average response and the "ribs" represent variations due to variations in mudcake density and composition.
- the bulk density is taken as equal to the porosity times the fluid density plus one minus the porosity times the rock matrix density: $\rho_b = \Phi\rho_f + (1 - \Phi)\rho_m$
  where ρ_b, ρ_f, and ρ_m are the bulk, fluid, and rock matrix densities, respectively, and Φ is the porosity of the rock.
- the main response is considered to be a simple exponential with no variation due to the photoelectric effect (rock composition) and no pre-exponential scattering term.
  the equation used is: $\Phi = \Phi_0 \exp[-\rho_b (Z/A) N_0 \sigma x]$

REVIEW OF EXISTING DENSITY LOG TREATMENT, 2

- Ellis defines the electron density index to be:

$$\rho_e=2(Z/A)\ \rho_b$$

- So that the tool response (or measured flux Φ) can be specified as proportional to:

$$\Phi \propto \exp(-\ \rho_e x)$$

- Ellis notes that the bulk density and electron density index are almost identical for the three most common rock minerals; calcite or limestone, $CaCO_3$, Quartz, SiO_2, and dolomite, $CaCO_3MgCO_3$, but both filling fluids of oil or water contain H with a Z/A≈1 so ρ_e is not exactly proportional to ρ_b.
- So the transform between the measured ρ_e and the log measured density, ρ_{log} is adjusted so that water and limestone are a linear relationship:

$$\rho_{log}=1.0704\ \rho_e\ -0.188$$

REVIEW OF EXISTING DENSITY LOG TREATMENT, 3

- Unfortunately, the Ellis treatment does not directly take into account the photoelectric effect (absorption) interaction, which is also very important and also does not take into account the scattering "in" that is also important.
- Ellis uses the simple exponential model:

$$N=N_0\exp(-\mu\rho x)$$

- At the time of the Ellis book "wireline" logs were being used exclusively and the main density measurement interference was mudcake from the drilling mud. The resulting treatment of this is shown in the following slides.

REVIEW OF EXISTING DENSITY LOG TREATMENT, 4

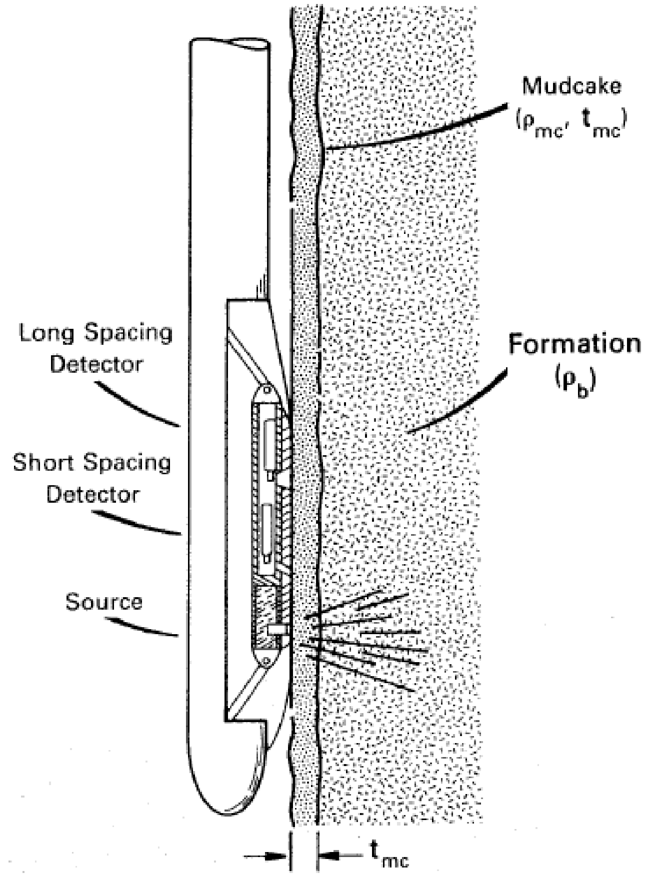

Figure 10-3. A formation density device in the borehole situation applied to the borehole wall and separated from it by the thickness of the mudcake, t_{mc}. From Ellis et al.[4]

REVIEW OF EXISTING DENSITY LOG TREATMENT, 5

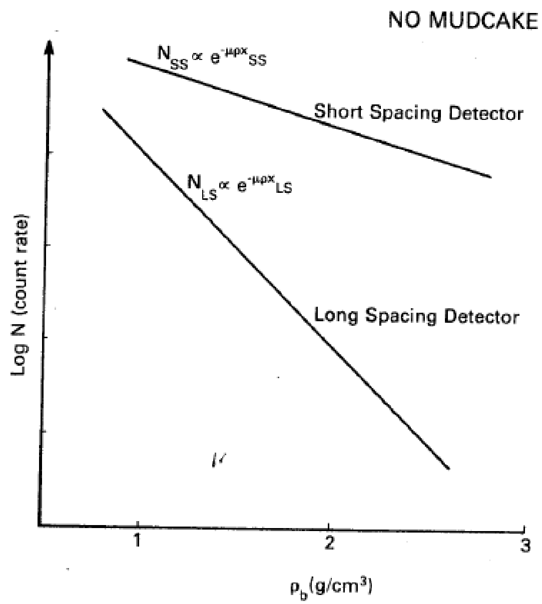

Figure 10-4. Idealized counting rate response of two detectors for variation formation density in the presence of no mudcake.

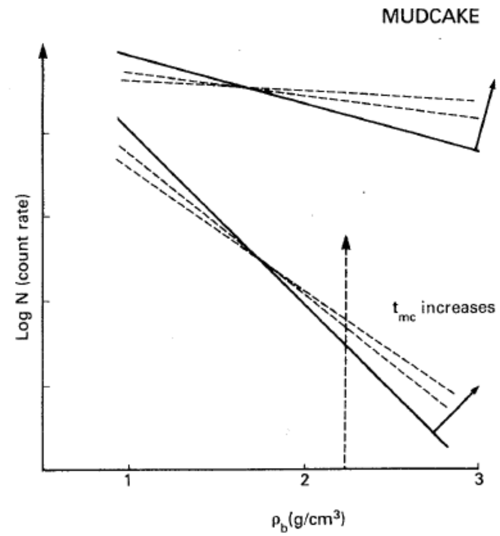

Figure 10-5. Idealized response of the two detectors for two different thicknesses of mudcake. Its density is approximately 1.8 g/cm³. For a given thickness of mudcake and formation density/mudcake density contrast, both detectors experience approximately the same percentage of change in counting rate.

REVIEW OF EXISTING DENSITY LOG TREATMENT, 6

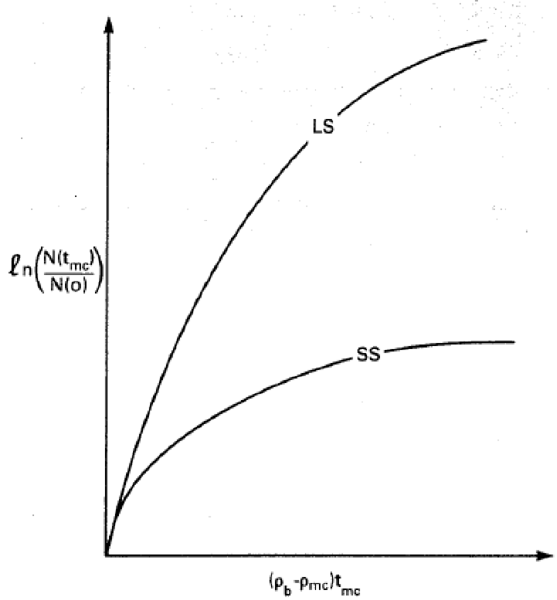

Figure 10-6. Effect of mudcake on the normalized counting rate of near and far detectors. The controlling parameter is the contrast in mudcake and formation density multiplied by the mudcake thickness.

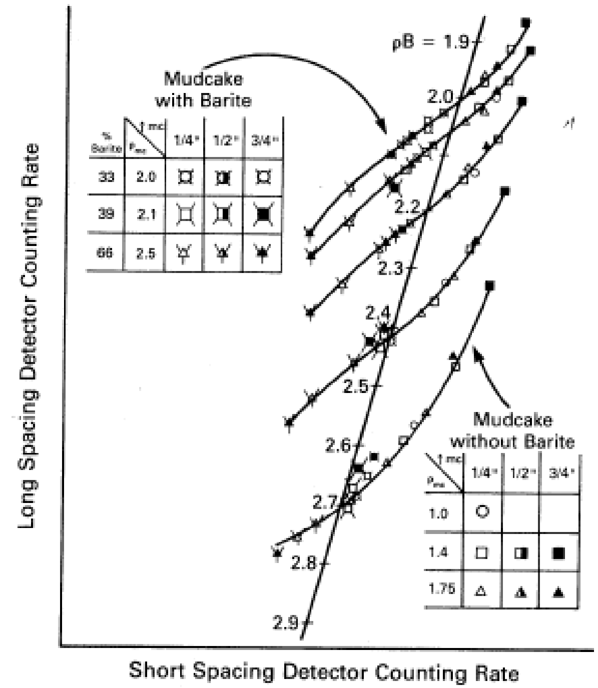

Figure 10–7. A "spine and ribs" representation of the response of a two-detector density device to formation density and mudcakes. Because of its characteristic outline, it is known as the "spine and ribs" chart. From Tittman et al.[3]

REVIEW OF EXISTING DENSITY LOG TREATMENT, 7

- Ellis next treats "lithology" logging by first defining a new parameter called the "photoelectric index", P_e, which is given by:

$$P_e=(Z/10)^{3.6}$$

- Then the attenuation relationship can be given as: $\Phi \propto \exp(-\, n_e P_e x)$

  where n_e is the number density of electrons as in the case of Compton scattering.

- The gamma-ray spectra taken with recent logs yields the "lithology" as shown in the following three figures.

REVIEW OF EXISTING DENSITY LOG TREATMENT, 8

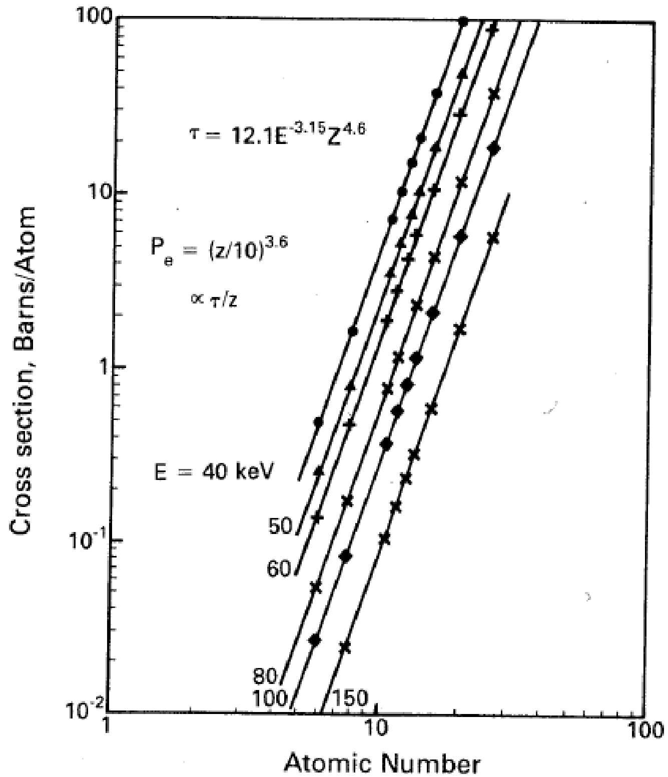

Figure 10–11. The variation of the photoelectric cross section with atomic number. From Bertozzi et al.[6]

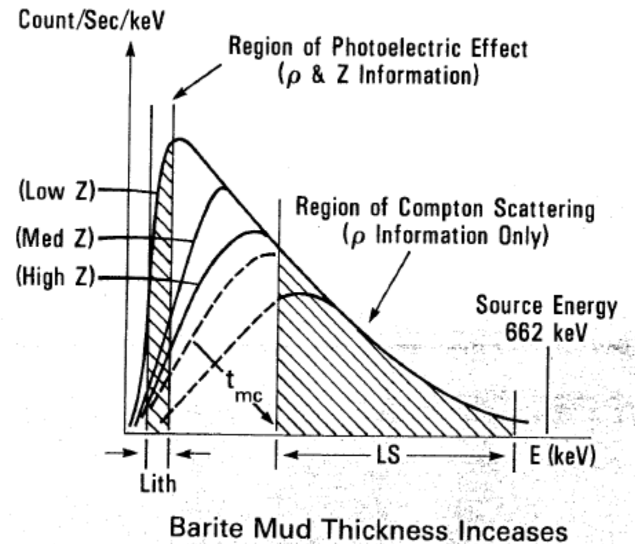

Figure 10–12. A schematic representation of the detected spectral variation of gamma rays in three formations of increasing average atomic number. Additional low energy absorption is noted for the presence and increasing thickness of absorbing barite mudcake. From Ellis et al.[4]

REVIEW OF EXISTING DENSITY LOG TREATMENT, 9

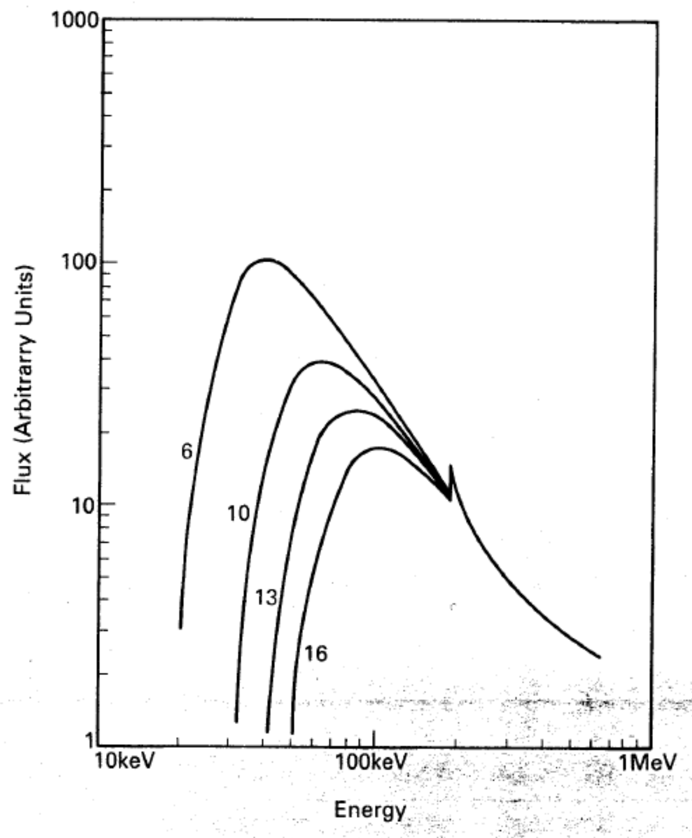

Figure 10–14. A Monte Carlo calculation of the spectrum due to uniformly distributed sources, including the effect of photoelectric absorption. The evolution of the spectra is seen for materials of four different atomic numbers. As the atomic number increases, the low energy portion of the spectrum is attenuated. From Bertozzi et al.[6]

THE FORWARD SCATTER MODEL APPROACH

- The basic response is taken as:

$$R=A_3X_S\exp(-A_1X_S-A_2X_A)+A_4$$

- Where the X_S and X_A are the scattering and absorption cross sections given by:

$$X_S= \rho\Sigma_i\, w_iZ_i/A_i$$

and $$X_A= \rho\Sigma_i\, w_iZ_i^{4.4}/A_i$$

- Note that the X_S term is essentially $\rho/2$ for all elements other than hydrogen. For the common oil well logging rocks the variation in Z/A is less than 1% when no hydrogen is present.

THE FORWARD SCATTER MODEL APPROACH, 2

- So if we express X_S in terms of either hydrogen and all other elements (or water or oil and all other elements), we then have:

  $X_S = 0.99209\ \rho w_H + 0.50000\ \rho(1 - w_H)$

  or $X_S = 0.50000\ \rho + 0.49209\ w_H\ \rho$

- Likewise we can express X_A in the same manner as:

  $X_A = (1^{4.4}/1.00729)\rho w_H + (1 - w_H)\ \rho\Sigma_i\ w_i Z_i^{4.4}/A_i\ (i>1)$

  or $X_A = 0.99209\ \rho w_H + (1 - w_H)\ \rho\Sigma_i\ w_i Z_i^{4.4}/A_i\ (i>1)$

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THE FORWARD SCATTER MODEL APPROACH, 3

- Now we can express the near and far detector responses as:

  $R_N=A_3X_S\exp(-A_1X_S-A_2X_A)+A_4$

  $R_F=B_3X_S\exp(-B_1X_S-B_2X_A)+B_4$

- These two equations can be solved simultaneously for X_A to give:

  $$X_A=A_1X_S/A_2+(1/A_2)\ln[(R_N-A_4)/(A_3X_S)]$$
  $$=B_1X_S/B_2+(1/B_2)\ln[(R_F-B_4)/(B_3X_S)]$$

- From which we can define $f(X_S)$ as:

  $$f(X_S)=0=(B_2A_1-A_2B_1)X_S+B_2\ln[(R_N-A_4)/(A_3X_S)] - A_2\ln[(R_F-B_4)/(B_3X_S)]$$

THE FORWARD SCATTER MODEL APPROACH, 4

- The differential of $f(X_S)$ wrt X_S is given by: $df(X_S)/dX_S=B_2A_1-A_2B_1-B_2/X_S+A_2/X_S$
- Now the root of X_S can be obtained by iterating on:

$$X_{Sn+1}=X_{Sn} - f(X_{Sn})/(df(X_{Sn})/dX_{Sn})$$

until the change in X_S is less than a specified value.

- This is called the Newton Iterative approach.

THE FORWARD SCATTER MODEL APPROACH, 5

- **Now the value of ρ can be obtained from:**
  X_S=0.50000 ρ +0.49209 w_H ρ
  or ρ=X_S/(0.50000+0.49209 w_H)
- **So if w_H is known – say from the porosity tool, then ρ is easily and accurately determined from the density tool measurement – using the near and far responses with this model.**
- **Now we can check to see if this works by using Monte Carlo simulation to generate R_N and R_F reponses.**

THE FORWARD SCATTER MODEL APPROACH, 6

- Thirteen responses each were generated by Monte Carlo simulation with MCNP5 for a range of densities (and corresponding water contents) of limestone, sandstone, and shale for both the near and far detectors of a density log.
- Then the model parameters were determined for both near and far detectors by using the data from all three types of rock for both the near and far detectors. The parameters obtained are given in the following table. Monte Carlo and model responses are shown plotted in the four following figures following the table.

MODEL VS. MONTE CARLO RESULTS – MODEL PARAMETERS

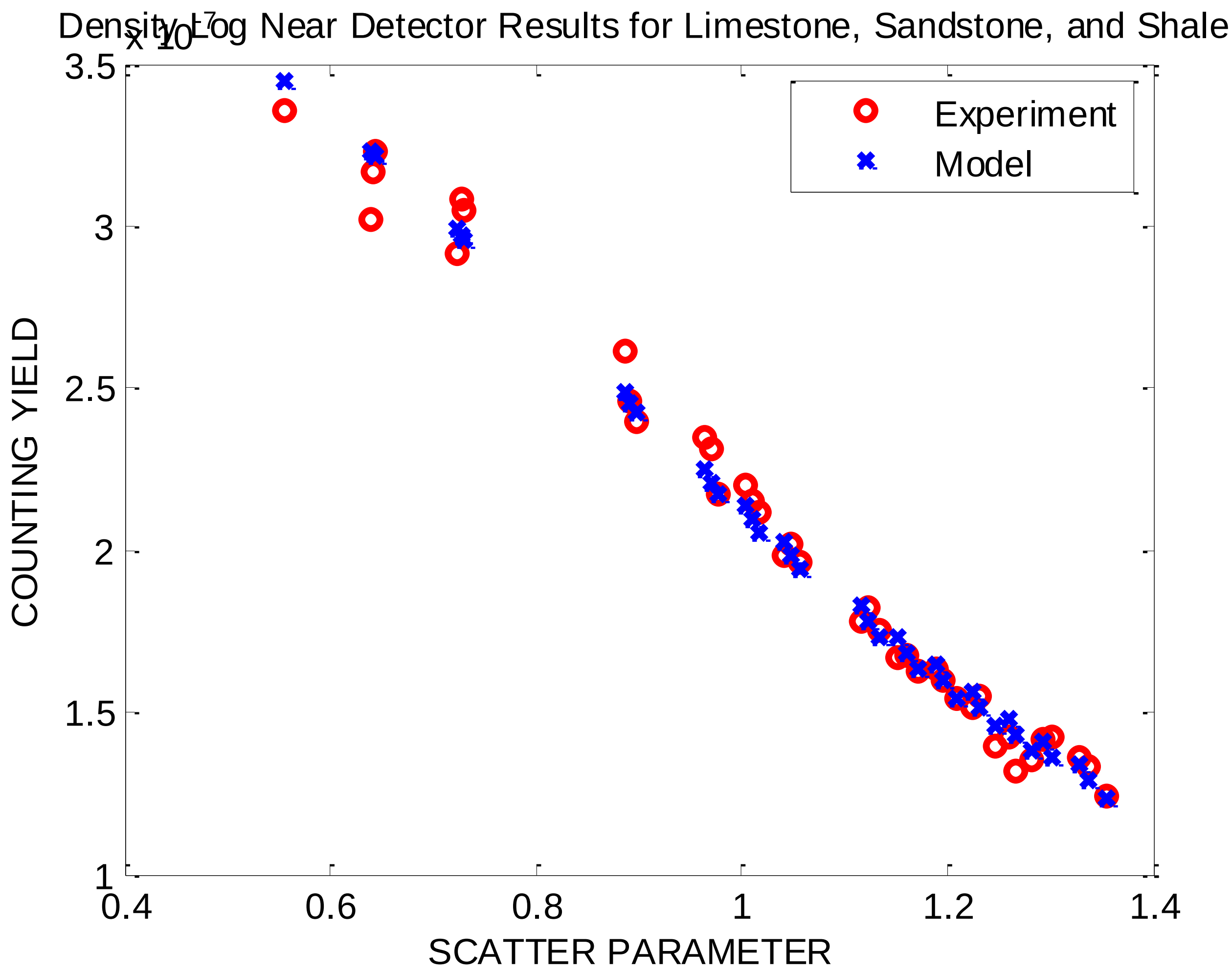
FAR (χ_η^2=1.46)

A_1=5.85081	$\sigma(A_1)$=0.01015
A_2=5.38332E-5	$\sigma(A_2)$=2.542E-6
A_3=1.06696E-4	$\sigma(A_3)$=5.458E-7
A_4=1.14036E-8	$\sigma(A_4)$=1.012E-9

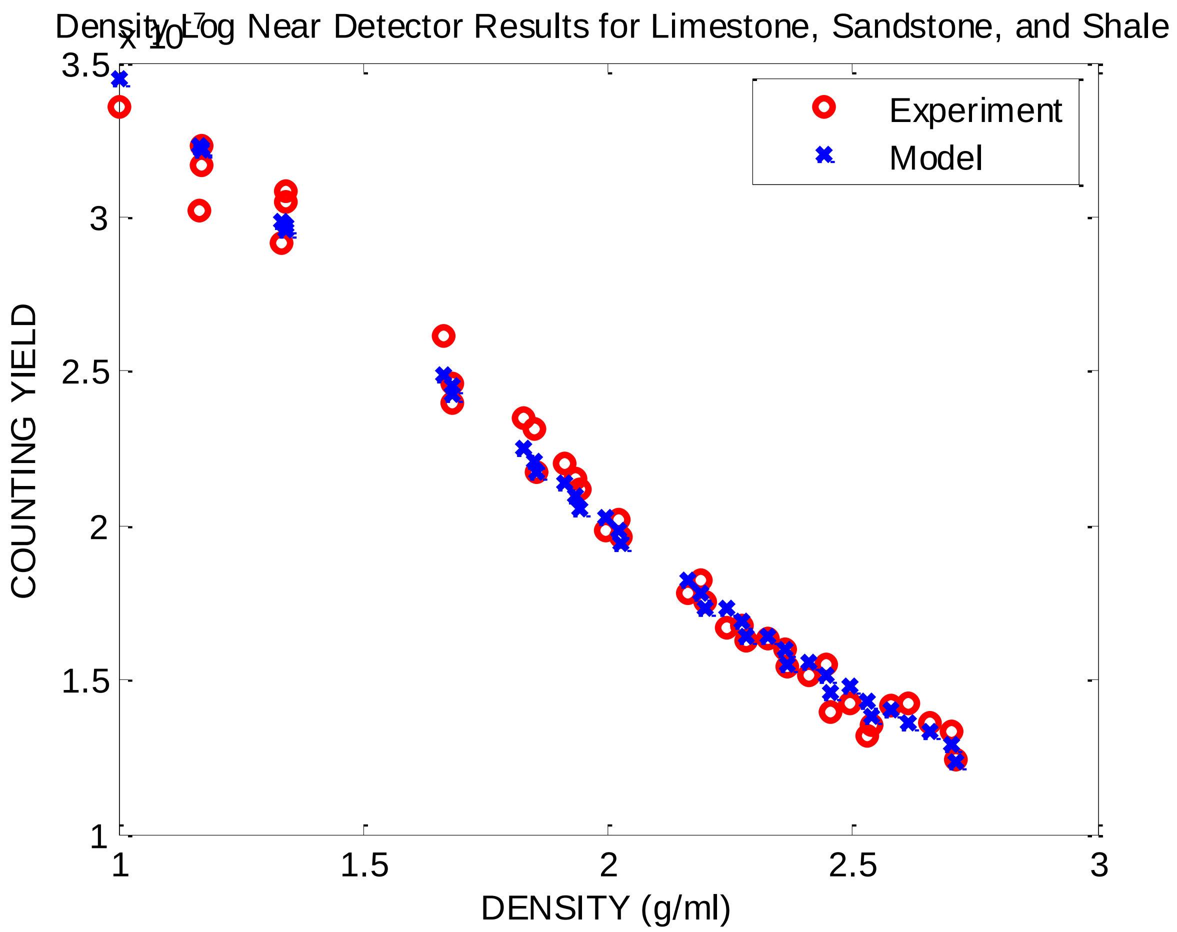
NEAR (χ_η^2=0.870)

B_1=2.44637	$\sigma(B_1)$=0.01386
B_2=5.44215E-6	$\sigma(B_2)$=2.408E-6
B_3=2.27403E-6	$\sigma(B_3)$=4.596E-8
B_4=2.02098E-8	σ(B4)=3.506E-9

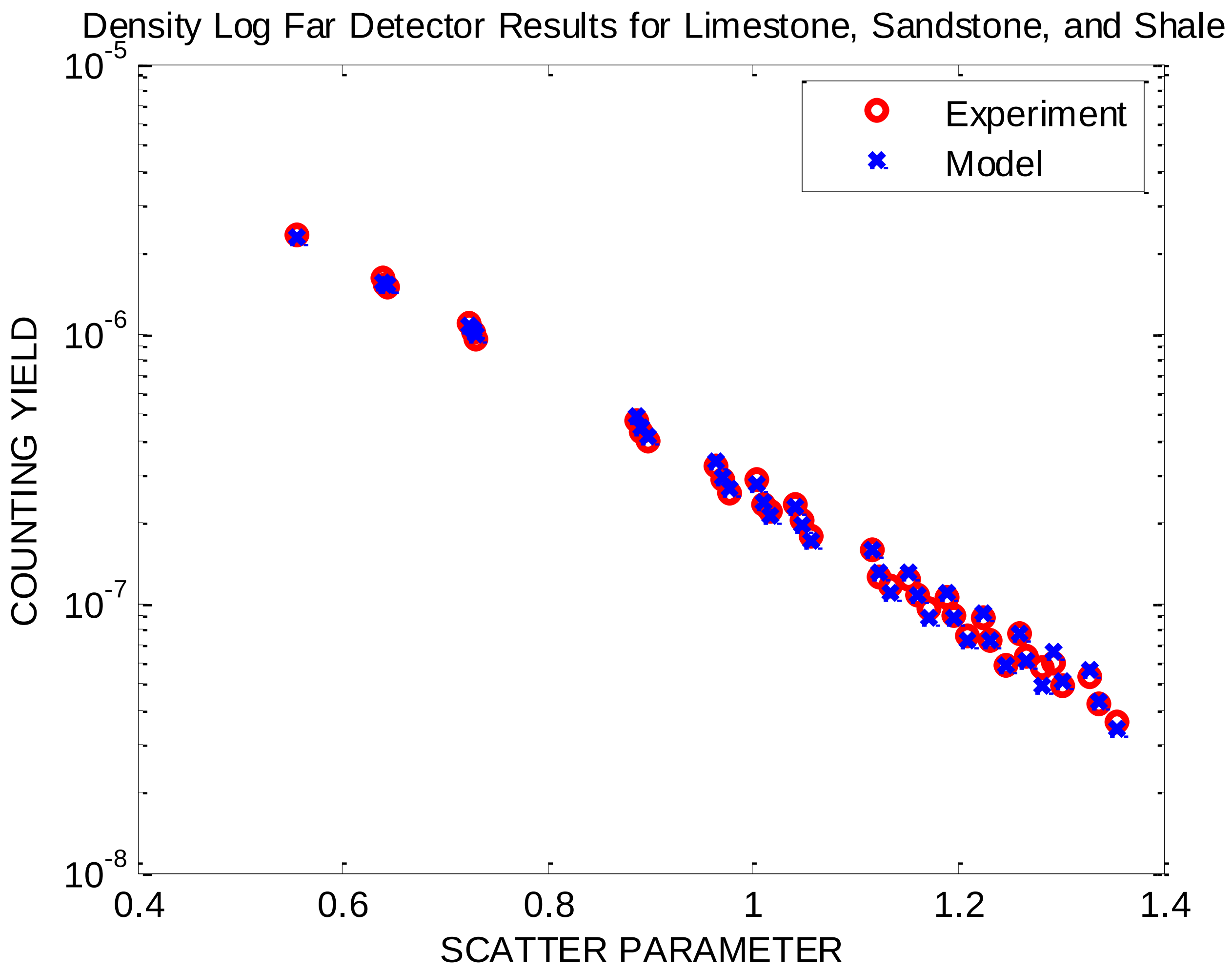
NEAR DETECTOR RESULTS



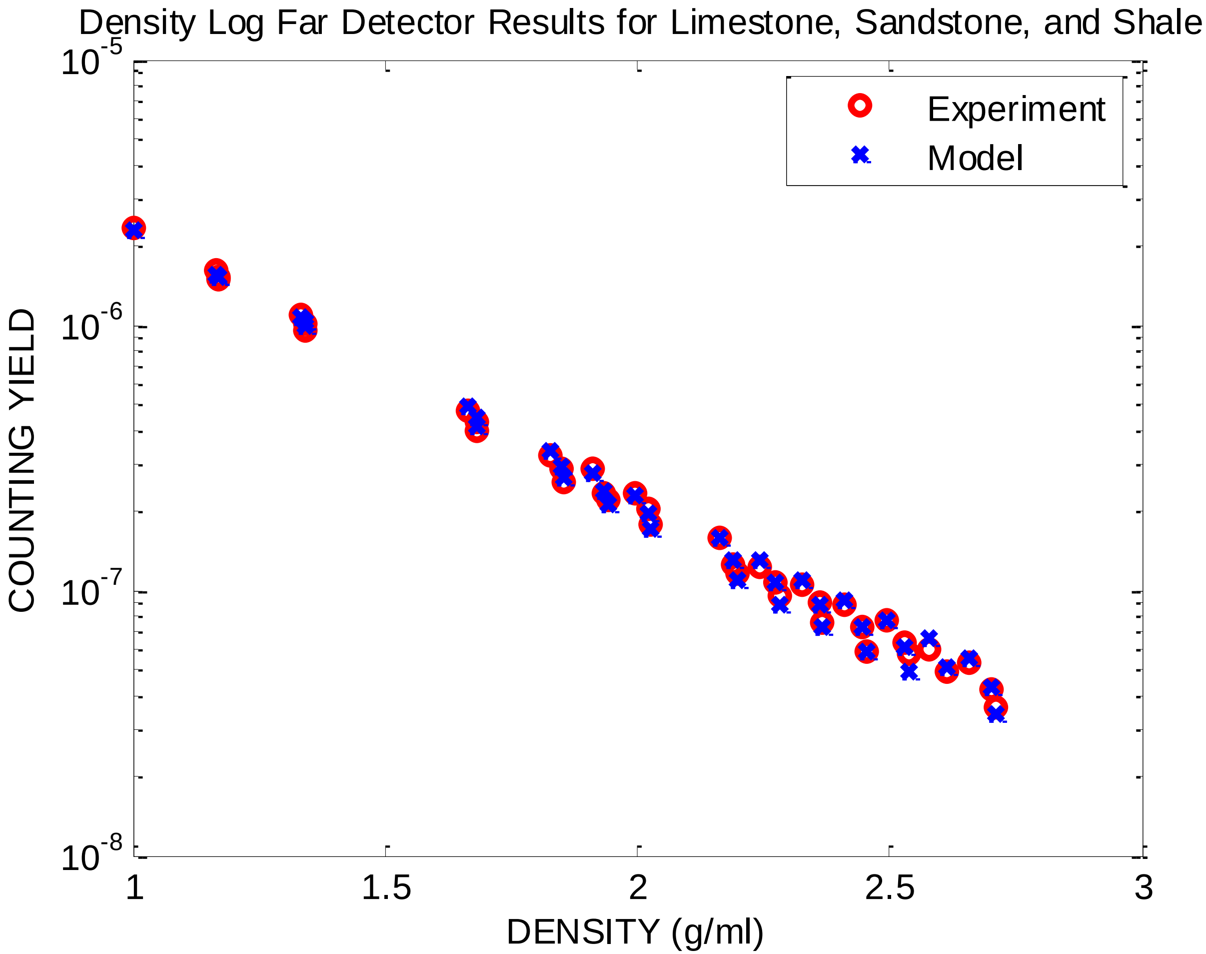
NEAR DETECTOR RESULTS



FAR DETECTOR RESULTS



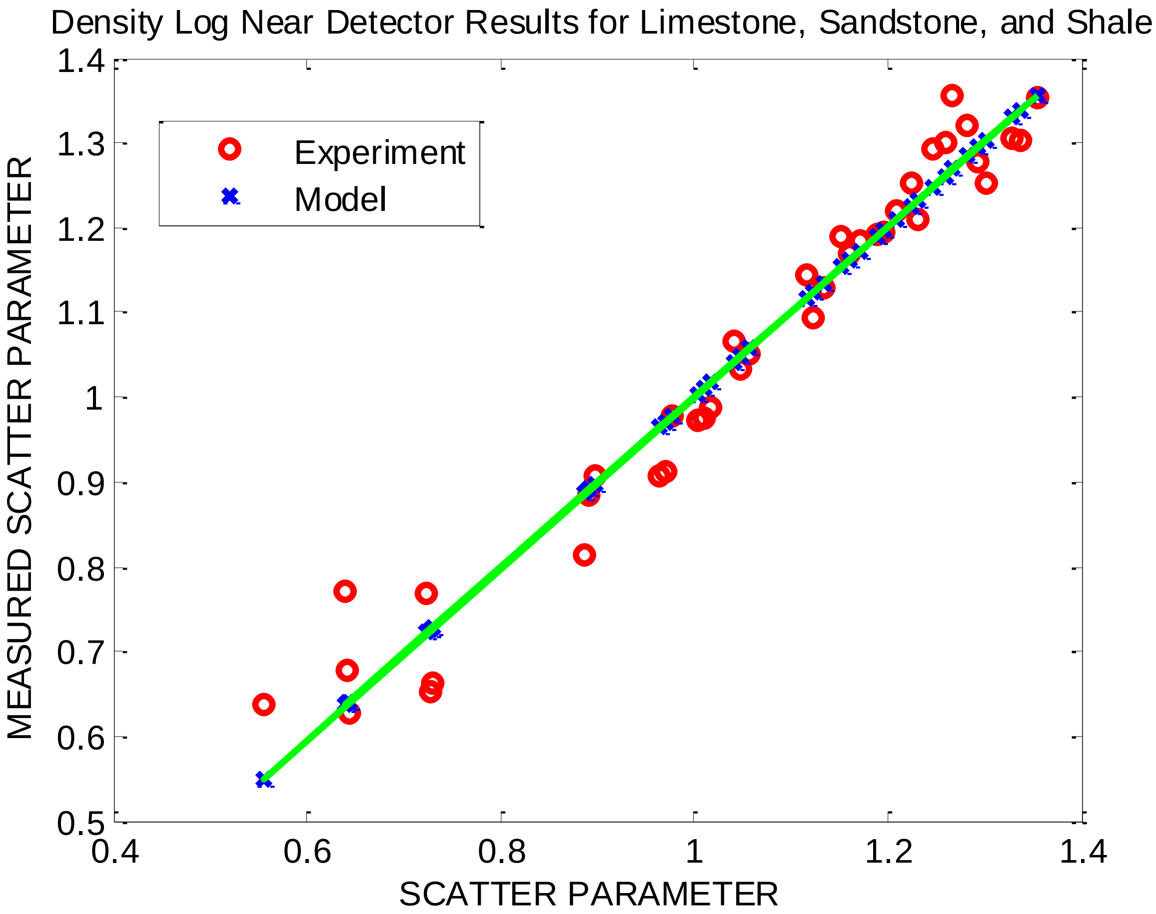
FAR DETECTOR RESULTS



THE FORWARD SCATTER MODEL APPROACH, 7

- The scatter parameter X_S is found by the method outlined on slide 15.
- The scatter parameters for both the experimental and model based values are plotted in the following figure. Note that the scatter parameters for the model based case give perfect results as expected, while those for the experimental case give quite a bit of variation. We believe this variation is caused primarily by the near detector calculations, which exhibit fairly large variations. Calculations for a larger number of histories should be made for the near detector case.

Determination of Scatter Parameter



DISCUSSION, FUTURE WORK, AND CONCLUSIONS

- The new approach has the major advantage that it can provide intuitive insight since it is a semi-empirical model rather than an empirical one and includes the effect of both scatter and absorption in an appropriate way.
- It is believed that the parameters that best describe the effect of scattering and absorption are X_S/ρ and X_A/ρ rather than ρ_e and P_E. The latter parameters include both density and the sum of Z/A and Z^n/A, while the new parameters separate the effect of composition and density.

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